

1. Solve the following equations:

$$\begin{aligned}
 1.1. \quad 2(x-1) - 3(x+1) &= 4(x-2) \Leftrightarrow 2x - 2 - 3x - 3 = 4x - 8 \\
 &\Leftrightarrow 2x - 3x - 4x = 5 - 8 \\
 &\Leftrightarrow -5x = -3 \\
 &\Leftrightarrow x = \frac{3}{5}
 \end{aligned}$$

Therefore:

$$x = \frac{3}{5} \quad \text{and} \quad S = \left\{ \frac{3}{5} \right\}$$

$$\begin{aligned}
 1.2. \quad 7(x+4) - 3(x+2) &= 7 - x \Leftrightarrow 7x + 28 - 3x - 6 = 7 - x \\
 &\Leftrightarrow 7x - 3x + x = 6 + 7 - 28 \\
 &\Leftrightarrow 5x = -15 \\
 &\Leftrightarrow x = -3
 \end{aligned}$$

Therefore

$$x = -3 \quad \text{and} \quad S = \{-3\}$$

$$\begin{aligned}
 1.3. \quad -7x - 3 &= 2\left(4 - \frac{1}{5}x\right) \Leftrightarrow -7x - 3 = 8 - \frac{2}{5}x \quad | \times 5 \\
 &\Leftrightarrow -35x + 2x = 15 + 40 \\
 &\Leftrightarrow -33x = 55 \\
 &\Leftrightarrow x = -\frac{5}{3}
 \end{aligned}$$

Therefore

$$x = -\frac{5}{3} \quad \text{and} \quad S = \left\{ -\frac{5}{3} \right\}$$

$$\begin{aligned}
 1.4. \quad \frac{1}{3}(x+2) - \frac{3}{4}(x-2) &= \frac{1}{12}(2-5x) + 2 \quad | \times 12 \Leftrightarrow 4(x+2) - 3 \cdot 3 \cdot (x-2) = (2-5x) + 2 \cdot 12 \\
 &\Leftrightarrow 4x + 8 - 9x + 18 = 2 - 5x + 24 \\
 &\Leftrightarrow 4x - 9x + 5x = 2 - 8 - 18 + 24 \\
 &\Leftrightarrow 0 = 0
 \end{aligned}$$

Therefore:

The equation is indeterminate and $S = \mathbb{R}$

$$\begin{aligned}
 1.5. \quad \frac{x+3}{3} - \frac{4x-3}{3} - 1 &= -\frac{5x-12}{6} \quad | \times 6 \Leftrightarrow 2 \cdot (x+3) - 2 \cdot (4x-3) - 6 = -(5x-12) \\
 &\Leftrightarrow 2x + 6 - 8x + 6 - 6 = -5x + 12 \\
 &\Leftrightarrow 2x - 8x + 5x = 12 - 6 \\
 &\Leftrightarrow -x = 6 \\
 &\Leftrightarrow x = -6
 \end{aligned}$$

Therefore:

$$x = -6 \quad \text{and} \quad S = \{-6\}$$

$$\begin{aligned}
1.6. \quad x\sqrt{2}-3(\sqrt{3}+\sqrt{2}) &= \sqrt{3}(1-x) \Leftrightarrow x\sqrt{2}-3\sqrt{3}-3\sqrt{2} = \sqrt{3}-x\sqrt{3} \\
&\Leftrightarrow x\sqrt{2}+\sqrt{3}x = 3\sqrt{3}+3\sqrt{2}+\sqrt{3} \\
&\Leftrightarrow x(\sqrt{2}+\sqrt{3}) = 4\sqrt{3}+3\sqrt{2} \\
&\Leftrightarrow x = \frac{4\sqrt{3}+3\sqrt{2}}{\sqrt{2}+\sqrt{3}} \\
&\Leftrightarrow x = \frac{(4\sqrt{3}+3\sqrt{2})(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})} \\
&\Leftrightarrow x = \frac{4\sqrt{6}-4\cdot 3+3\cdot 2-3\sqrt{6}}{2-3} \\
&\Leftrightarrow x = -\sqrt{6}+6
\end{aligned}$$

Therefore:

$$x = -\sqrt{6}+6 \quad \text{and} \quad S = \{-\sqrt{6}+6\}$$

2. Solve the following equations:

$$\begin{aligned}
2.1. \quad (2x-3)^2-(x+5)^2 &= 0 \Leftrightarrow ((2x-3)-(x+5)) \cdot ((2x-3)+(x+5)) = 0 \\
&\Leftrightarrow (2x-3-x-5) \cdot (2x-3+x+5) = 0 \\
&\Leftrightarrow (x-8) \cdot (3x+2) = 0 \\
&\Leftrightarrow \begin{cases} x-8=0 \\ \text{ou} \\ 3x+2=0 \end{cases} \\
&\Leftrightarrow \begin{cases} x=8 \\ \text{ou} \\ x=-\frac{2}{3} \end{cases}
\end{aligned}$$

Therefore

$$x = 8 \text{ ou } x = -\frac{2}{3} \quad \text{and} \quad S = \left\{-\frac{2}{3}; 8\right\}$$

$$\begin{aligned}
2.2. \quad 9(2x-1)^2 &= 4(x-2)^2 \Leftrightarrow 9(2x-1)^2 - 4(x-2)^2 = 0 \\
&\Leftrightarrow (3(2x-1)-2(x-2)) \cdot (3(2x-1)+2(x-2)) = 0 \\
&\Leftrightarrow (6x-3-2x+4) \cdot (6x-3+2x-4) = 0 \\
&\Leftrightarrow (4x+1) \cdot (8x-7) = 0
\end{aligned}$$

$$(4x+1) \cdot (8x-7) = 0 \Leftrightarrow \begin{cases} x = -\frac{1}{4} \\ \text{ou} \\ x = \frac{7}{8} \end{cases}$$

Therefore

$$x = -\frac{1}{4} \text{ ou } x = \frac{7}{8} \quad \text{and} \quad S = \left\{-\frac{1}{4}; \frac{7}{8}\right\}$$

$$\begin{aligned}
2.3. \quad 81x^2 - 64 - (9x+8)(2x+7) = 0 &\Leftrightarrow (81x^2 - 64) - (9x+8)(2x+7) = 0 \\
&\Leftrightarrow (81x^2 - 64) - (9x+8)(2x+7) = 0 \\
&\Leftrightarrow (9x-8)(9x+8) - (9x+8)(2x+7) = 0 \\
&\Leftrightarrow (9x+8) \cdot [(9x-8) - (2x+7)] = 0 \\
&\Leftrightarrow (9x+8) \cdot [9x-8-2x-7] = 0 \\
&\Leftrightarrow (9x+8) \cdot (7x-15) = 0 \\
&\Leftrightarrow \begin{cases} 9x+8=0 \\ \text{ou} \\ 7x-15=0 \end{cases} \\
&\Leftrightarrow \begin{cases} x = -\frac{8}{9} \\ \text{ou} \\ x = \frac{15}{7} \end{cases}
\end{aligned}$$

Therefore

$$x = -\frac{8}{9} \text{ ou } x = \frac{15}{7} \text{ and } S = \left\{ -\frac{8}{9}; \frac{15}{7} \right\}$$

$$\begin{aligned}
2.4. \quad x(2x-3) + (2x-3) = (x-3)(2x-3) &\Leftrightarrow x(2x-3) + (2x-3) - (x-3)(2x-3) = 0 \\
&\Leftrightarrow (2x-3) \cdot [x+1 - (x-3)] = 0 \\
&\Leftrightarrow (2x-3) \cdot (x+1-x+3) = 0 \\
&\Leftrightarrow (2x-3) \cdot 4 = 0 \\
&\Leftrightarrow 2x-3 = 0 \\
&\Leftrightarrow x = \frac{3}{2}
\end{aligned}$$

Therefore

$$x = \frac{3}{2} \text{ and } S = \left\{ \frac{3}{2} \right\}$$