Christmas Examination

Monday 15 Dec. 2025

Duration : 90 min

Maths SL IB$_2$

Part 1

(8 Problems 83 marks)

Name: __________

A calculator is *not* allowed for this fisrt part

Problem 1

[/ 4 marks]

Consider the function $f(x)=x^2+5x-8$, where $x\in\mathbb{R}$

(a) Find $f'(1)$. [2]

(b) Find the equation of the tangent to the graph of f at $x=1$. [2]

Problem 2

[/ 4 marks]

The derivative of a function g is given by $g'(x)=\cos x+\mathrm{e}^{2x}$, where $x\in\mathbb{R}$.

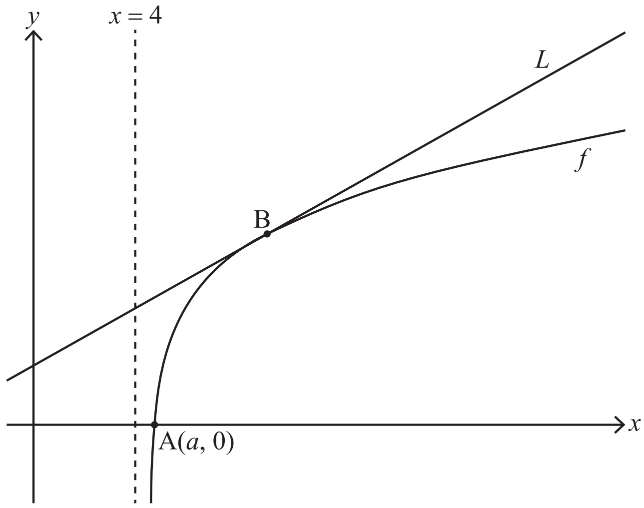
Given that $g(0)=7$, find $g(x)$.

Problem 3

[/ 9 marks]

Consider the function f defined by $f(x)=\ln(x^2-16)$ for $x>4$.

The following diagram shows part of the graph of f which crosses the x-axis at point A, with coordinates $(a,0)$. The line L is the tangent to the graph of f at the point B.



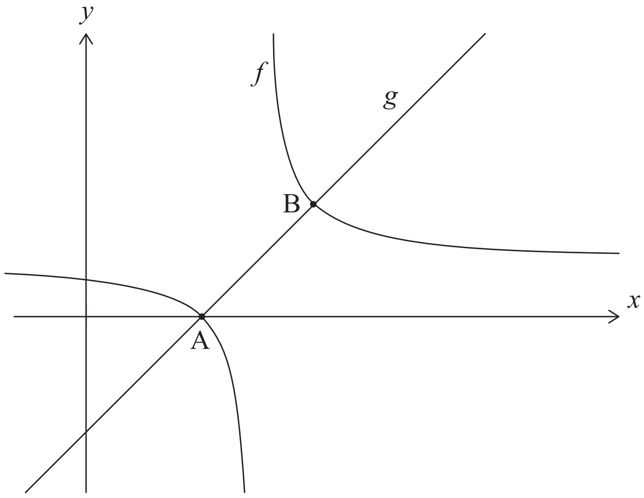
(a) Find the exact value of a. [3]

(b) Given that the gradient of L is $\frac{1}{3}$, find the x-coordinate of B. [6]

Problem 4 [/ 15 marks]

Consider the functions $f(x) = \frac{1}{x-4} + 1$, for $x \neq 4$, and $g(x) = x - 3$ for $x \in \mathbb{R}$.

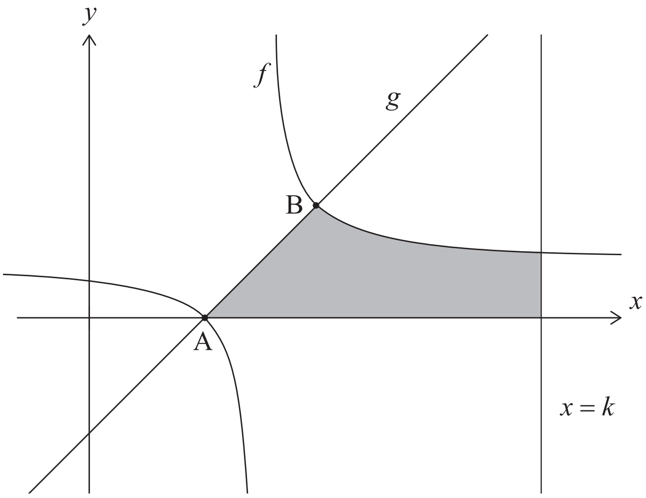
The following diagram shows the graphs of f and g.



The graphs of f and g intersect at points A and B. The coordinates of A are $(3, 0)$.

(a) Find the coordinates of B. [5]

In the following diagram, the shaded region is enclosed by the graph of f, the graph of g, the x-axis, and the line $x = k$, where $k \in \mathbb{Z}$.



The area of the shaded region can be written as $\ln(p) + 8$, where $p \in \mathbb{Z}$.

(b) Find the value of k and the value of p. [10]

Problem 5 [/ 17 marks]

The function f is defined by $f(x) = 4^x$, where $x \in \mathbb{R}$.

(a) Find $f^{-1}(8)$. Express your answer in the form $\frac{p}{q}$ where $p, q \in \mathbb{Z}$. [3]

The function g is defined by $g(x) = 1 + \log_2 x$, where $x \in \mathbb{R}^+$.

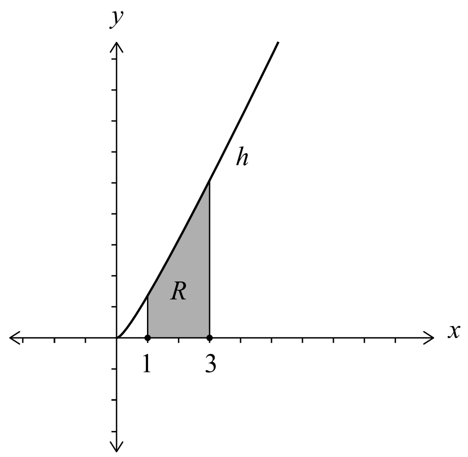
(b) (i) Find an expression for $g^{-1}(x)$.

(ii) Describe a sequence of transformations that transforms the graph of $y = g^{-1}(x)$ to the graph of $y = f(x)$. [4]

(c) Show that $(f \circ g)(x) = 4x^2$. [3]

The function h is defined by $h(x) = \frac{4x^2}{2x+1}$, $x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of h. Let R be the region enclosed by the graph of h and the x-axis, between the lines $x = 1$ and $x = 3$.



(d) (i) Show that $2x - 1 + \frac{1}{2x+1} = \frac{4x^2}{2x+1}$.

(ii) Hence or otherwise, find the area of R, giving your answer in the form $p + q\ln r$, where $p, q, r \in \mathbb{Q}^+$. [7]

Problem 6 [/ 16 marks]

A particle P moves along the x-axis. The velocity of P is $v\,\text{m}\,\text{s}^{-1}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \leq t \leq 3$. When $t = 0$, P is at the origin O.

(a) (i) Find the value of t when P reaches its maximum velocity.

(ii) Show that the distance of P from O at this time is $\frac{88}{27}$ metres. [7]

(b) Sketch a graph of v against t, clearly showing any points of intersection with the axes. [4]

(c) Find the total distance travelled by P. [5]

Problem 7 [/ 5 marks]

Box 1 contains 5 red balls and 2 white balls.
Box 2 contains 4 red balls and 3 white balls.

(a) A box is chosen at random and a ball is drawn. Find the probability that the ball is red. [3]

Let A be the event that "box 1 is chosen" and let R be the event that "a red ball is drawn".

(b) Determine whether events A and R are independent. [2]

Problem 8 [/ 13 marks]

A discrete random variable, X, has the following probability distribution, where $a > 0$ and k is a constant.

x	0	a	$2a$	$3a$
$\mathrm{P}(X = x)$	k	$3k^2$	$2k^2$	k^2

(a) Show that $k = \frac{1}{3}$. [5]

(b) Find $\mathrm{P}(X < 3a)$. [2]

(c) Find $\mathrm{P}(X \geq a \mid X < 3a)$. [3]

(d) Given that $\mathrm{E}(X) = 20$, find the value of a. [3]