Christmas Examination

Maths SL IB_2

Part 2

(8 Problems 52 marks)

Monday 15 Dec. 2025

Duration : 90 min

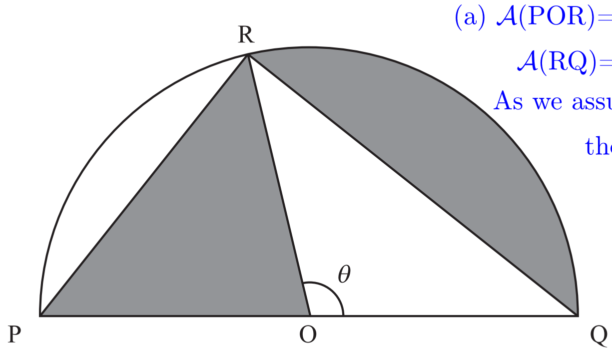
ANSWERS

A calculator **is allowed** for this second part

Problem 1

[/ 6 marks]

The following diagram shows a semicircle with centre O and radius r. Points P, Q and R lie on the circumference of the circle, such that $\mathrm{PQ} = 2r$ and $\mathrm{R\hat{O}Q} = \theta$, where $0 < \theta < \pi$.



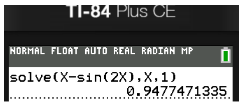
(a) $\mathcal{A}(\mathrm{POR}) = \frac{r^2 \sin(\theta)}{2}$

$\mathcal{A}(\mathrm{RQ}) = \frac{1}{2} r^2 \theta - \frac{r^2 \sin(\theta)}{2}$

As we assume $\mathcal{A}(\mathrm{POR}) = \mathcal{A}(\mathrm{RQ})$

then $2 \frac{r^2 \sin(\theta)}{2} = \frac{1}{2} r^2 \theta$

$\Rightarrow \theta = 2\sin(\theta)$



(a) Given that the areas of the two shaded regions are equal, show that $\theta = 2\sin\theta$. [5]

(b) Hence determine the value of θ. using a calculator (only !) : $\theta \cong 0.947\,\mathrm{rad} \cong 54.3^o$ [1]

Problem 2

[/ 5 marks]

Geometric sequence with $u_1 = 50$ $u_4 = 864$ hence $r^3 = \frac{86.4}{50} \Rightarrow r = 1.2$

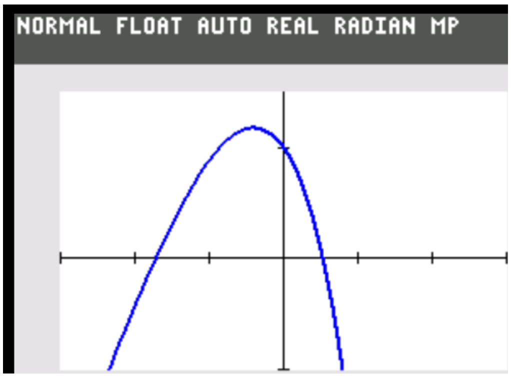
$S_n = u_1 \frac{1-r^n}{1-r} > 33500 \Leftrightarrow 50 \frac{1-r^n}{1-r} > 33500 \Leftrightarrow \frac{1-r^n}{1-r} > 670 \Leftrightarrow \frac{1-r^n}{1-r} > 670 \Leftrightarrow 1 - r^n > 670(1-r)$

$\Leftrightarrow r^n > 1 + 670(0.2) = 135$ $\log_{1.2}(135) = 26.9$ $\Rightarrow \boxed{n = 27}$

Problem 3 [/ 6 marks]

$f'(x) = 4 + 2x - 3e^x$ where $x \in \mathbb{R}$

(a) f increasing $\Leftrightarrow f'(x) > 0$



The figure shows the curve of $y = f'(x)$.

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solve(4+2X-3e^X,X,1)
                    0.5179997139
solve(4+2X-3e^X,X,-2)
                     -1.73554346
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Therefor $\boxed{f \text{ increasing for } -1.7355 < x < 0.518}$

(b) $f''(x) = 2 - 3e^x$ the curve of f is concave-up if $f''(x) < 0$

$\Leftrightarrow 2 - 3e^x < 0$

$\Leftrightarrow 2 < 3e^x$ $\Leftrightarrow e^x > \frac{2}{3}$ $\Leftrightarrow$ $\boxed{x > \ln\left(\frac{2}{3}\right) = -0.405}$

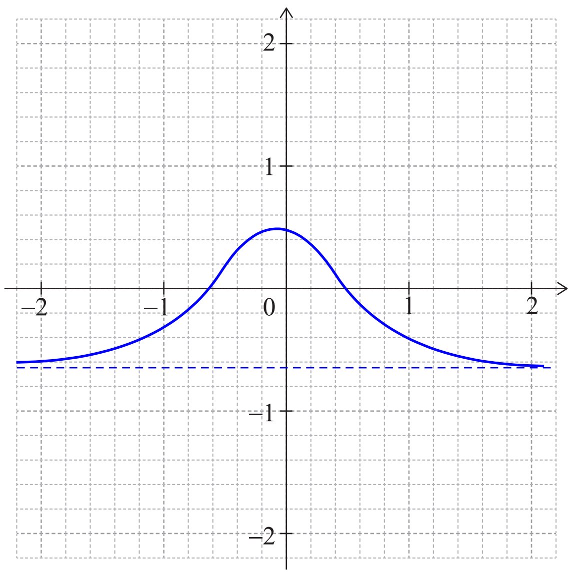
Problem 4 [/ 5 marks]

Consider the function $f(x) = e^{-x^2} - 0.5$, for $-2 \le x \le 2$.

(a) Find the values of x for which $f(x) = 0$. [2]

(b) Sketch the graph of f on the following grid. [3]

(b) graph



(a) $f(x) = 0$

$\Leftrightarrow e^{-x^2} = \frac{1}{2}$

$\Leftrightarrow -x^2 = -\ln(2)$

$\Leftrightarrow x = \sqrt{\ln(2)}$

$\cong 0.83$

Problem 5 [/ 7 marks]

$f(x) = \frac{(2x+a)^3}{(x+5)^2}$ where $x \neq -5$ and $a \in \mathbb{R}^+$.

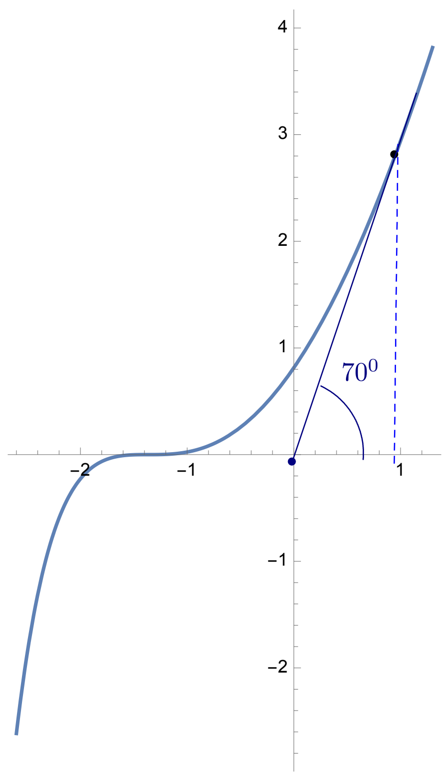
(a) $$f'(x) = \frac{3(2x+a)^2 \cdot 2 \cdot (x+5)^2 - (2x+a)^3 \cdot 2 \cdot (x+5)}{((x+5)^2)^2} = \frac{6(2x+a)^2(x+5)^2 - 2(2x+a)^3 \cdot (x+5)}{(x+5)^4}$$
$$= \frac{6(2x+a)^2(x+5) - 2(2x+a)^3}{(x+5)^3} = \frac{(2x+a)^2(6(x+5) - 2(2x+a))}{(x+5)^3} = \boxed{\frac{(2x+a)^2(2x+30-2a)}{(x+5)^3}}$$

(b) The two possible values of a for having a tangent at $x = 1$ with an angle of 70^0

are solutions of $f'(x) = \tan(70) \cong 2.75$

Then $(2x+a)^2(2x+30-2a) = 2.75(x+5)^3$ when $x = 1$

that is : $(2+a)^2(32-2a) = 2.75 \cdot 6^3 = 593.5$ $\Rightarrow \boxed{a = 2.7306}$

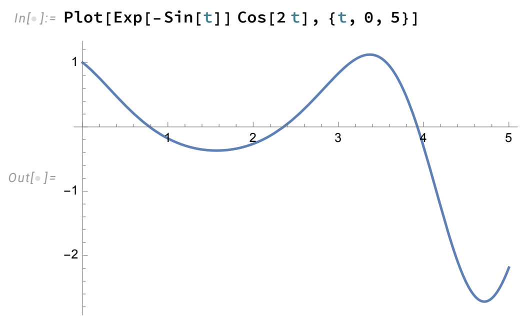


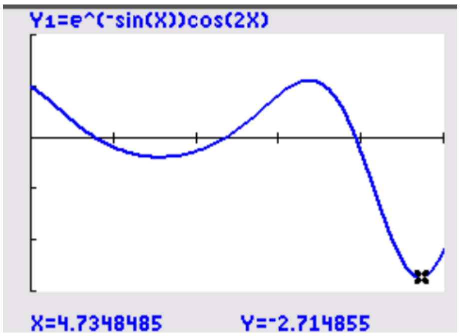
Problem 6 [/ 7 marks]

A particle P moves in a line. Its velocity is $v(t) = e^{-\sin(t)}\cos(2t)$ for $0 \leqslant t \leqslant 5\,s$.

(a) The maximum speed of P is solution of $\frac{dv}{dt} = 0$

But first let us look how is the curve of $v(t)$:





As we want the maximum speed (speed is absolute value of velocity)

we can see that its correspond to the minimum velocity. That is for t neat to 4.71 s.

We can use the solver (here $Y_1 = v(x)$) to find the zero of the derivative

$\text{solve}\left(\frac{d}{dX}(Y_1)\big|_{X=x}, X, 1\right)$ 1.570796327

$\text{solve}\left(\frac{d}{dX}(Y_1)\big|_{X=x}, X, 3\right)$ 3.368273513

$\text{solve}\left(\frac{d}{dX}(Y_1)\big|_{X=x}, X, 5\right)$ 4.71238898

According to the graph, only the last result is the correct one : maximum speed for $\boxed{t = 4.7124\,s}$

At that time : Y1(4.71238898) -2.718281828 $\Rightarrow$max speed is $\boxed{2.7173 \text{ ms}^{-1}}$

(b) Total distance travelled = $\int_0^5 |v(t)dt|$

$\int_0^5 (|Y_1|)dX$ 3.845920022

(c) The acceleration chamges *for the second time* near $t = 2, 3\,\text{sec}$, more precisely:

solve(Y1,X,2) 2.35619449

At that particular time, the acceleration is $\boxed{a = 0.986\,\text{ms}^{-2}}$

$\frac{d}{dX}(Y_1)\big|_{X=2.35619449}$ 0.9861373203

Problem 7 [/ 8 marks]

State two conditions required for X to be modeled by a binomial distribution.

Fixed number of trials

Each trial has *two* possible outcomes

The outcome of each trial is *independent* of all the others (*constant* probability of success)

(b) $1900 \times 0.37 = \boxed{703 \text{ people}}$

(c) (i) Let D be the number of people who will ride *Daifong*

$\mathrm{P}(D = 712) = \boxed{0.0172556...}$

(ii) $P(D \leqslant 712)$ $\underline{=0.674739...}$

$P(D \leqslant 683)$ $=0.177146...$

$\mathrm{P}(684 \leqslant D \leqslant 712) = \boxed{0.497593...}$

That would be interesting to continue this question, looking at Paper 2 May 2025 tz3

(d) $\mathrm{P}(\leqslant D \leqslant 692 | 684 \leqslant D \leqslant 712) = \frac{\mathrm{P}(684 \leqslant D \leqslant 692)}{\mathrm{P}(684 \leqslant D \leqslant 712)} = \frac{0.132318}{0.497593} = \boxed{0.266}$

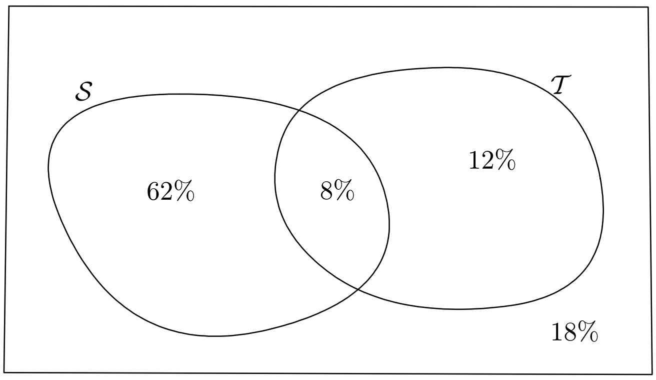
Problem 8 [/ 8 marks]

a) See the diagram

b) $\mathrm{P}(\mathcal{S} \cap \mathcal{T}^*) = 12\%$

c) $\mathrm{P}(\mathcal{G} \cap \mathcal{T}) = P(\mathcal{G}) P(\mathcal{T} | \mathcal{G})$
$= 0.48 \times 0.25 = 12\%$

d) $\mathrm{P}(\mathcal{T} | \mathcal{G}) \neq \mathrm{P}(\mathcal{T}) \Rightarrow$ *not* independent.
$\mathrm{P}(\mathcal{T} \cap \mathcal{G}) \neq P(\mathcal{T}) P(G) \Rightarrow$ *not* indep!



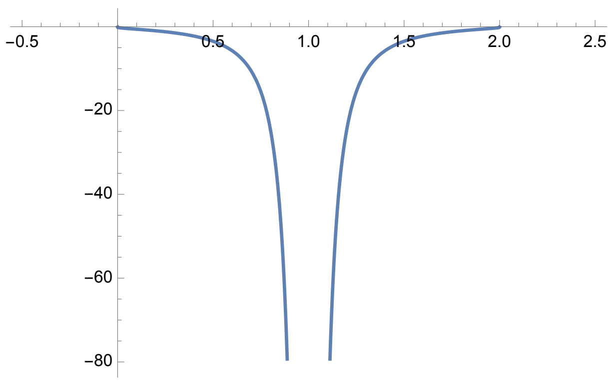
Bonus [+ 6]

Let us consider the function

$$f(x) = \frac{1}{\ln(2x - x^2)}$$

i) The *domain* of f is $\mathbb{R} \backslash \{1\}$

ii) The curve of equation $y = f(x)$ is shown ...



iii) The equation of the *vertical* asymptote: $\boxed{x = 1}$

iv) The equation of the horizontal asymptote: $\boxed{y = 0}$