Christmas Examination

Monday 15 Dec. 2025
Duration : 90 min

Maths SL IB_2

Part 2

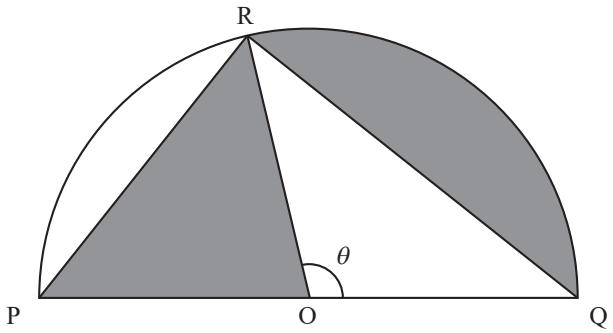
(8 Problems 52 marks)

Name : _____

A calculator **is allowed** for this second part

Problem 1 [/ 6 marks]

The following diagram shows a semicircle with centre O and radius r. Points P, Q and R lie on the circumference of the circle, such that $PQ = 2r$ and $R\hat{O}Q = \theta$, where $0 < \theta < \pi$.



(a) Given that the areas of the two shaded regions are equal, show that $\theta = 2\sin\theta$. [5]

(b) Hence determine the value of θ. [1]

Problem 2 [/ 5 marks]

A geometric sequence has a first term of 50 and a fourth term of 86.4.

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\,500$.

Problem 3 [/ 6 marks]

The derivative of a function f is given by $f'(x) = 4 + 2x - 3e^x$, where $x \in \mathbb{R}$.

(a) Find the values of x for which f is decreasing. [3]

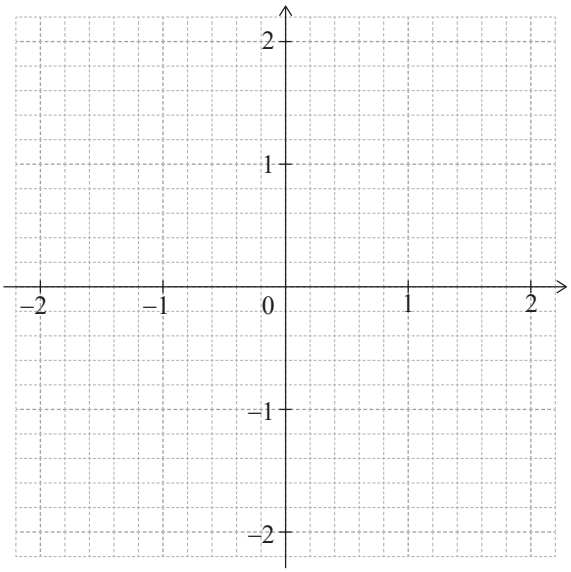
(b) Find the values of x for which the graph of f is concave-up. [3]

Problem 4 [/ 5 marks]

Consider the function $f(x) = e^{-x^2} - 0.5$, for $-2 \le x \le 2$.

(a) Find the values of x for which $f(x) = 0$. [2]

(b) Sketch the graph of f on the following grid. [3]



Problem 5 [/ 7 marks]

Consider the function $f(x) = \dfrac{(2x+a)^3}{(x+5)^2}$, where $x \neq -5$ and $a \in \mathbb{R}^+$.

(a) Find an expression for $f'(x)$, in terms of a. [3]

When $x = 1$, the tangent to the graph of f makes an angle of $70°$ to the horizontal.

(b) Find the two possible values of a. [4]

Problem 6 [/ 7 marks]

A particle P moves in a straight line. The velocity $v\,\text{m s}^{-1}$ of P, at time t seconds is given by $v(t) = e^{-\sin t}\cos(2t)$, for $0 \le t \le 5$.

(a) Find the maximum speed of P. [2]

(b) Find the total distance travelled by P. [2]

(c) Find the acceleration when P changes direction for the **second** time. [3]

Problem 7 [/ 8 marks]

(a) State two conditions required for X to be modelled by a binomial distribution. [2]

A water theme park has two rides: *Daifong* and *Torbellino*. Each visitor's decision to ride on either *Daifong* or *Torbellino* is made independently of any other person.

From previous records, it is expected that 37% of the visitors on any particular day will ride *Daifong*.

On Saturday, 1900 people will visit the theme park.

(b) Find the number of people that are expected to ride *Daifong*. [2]

(c) Find the probability that

(i) 712 people will ride *Daifong*;

(ii) between 684 and 712 people, inclusive, will ride *Daifong*. [4]

Problem 8 [/ 8 marks]

At a school, 70% of the students play a sport and 20% of the students are involved in theatre. 18% of the students do neither activity.

A student is selected at random.

(a) Find the probability that the student plays a sport and is involved in theatre. [2]

(b) Find the probability that the student is involved in theatre, but does not play a sport. [2]

At the school 48% of the students are girls, and 25% of the girls are involved in theatre.

A student is selected at random. Let G be the event "the student is a girl" and let T be the event "the student is involved in theatre".

(c) Find $\mathrm{P}(G \cap T)$. [2]

(d) Determine if the events G and T are independent. Justify your answer. [2]

Bonus [+ 6]

Let us consider the function

$$f(x) = \frac{1}{\ln(2x - x^2)}$$

i) Give the domain of f

ii) Draw the curve of equarion $y = f(x)$

iii) Give the equation of the vertical asymptote.