

Question 1Find the *domain* of the following functions

1) $f_1(x) = \frac{3x^5 - 17x^3 + x(x+1) - 2}{12}$

$D_1 = \mathbb{R}$

2) $f_2(x) = \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}$

$D_2 = \mathbb{R} \setminus \{4, 5, 6\}$

3) $f_3(x) = \frac{\sqrt{x}}{(x-2)(x-3)}$

$D_3 = [0, \infty[\setminus \{2, 3\}$

4) $f_4(x) = \frac{\sqrt{x-4}}{x^2 - 5x + 6}$

$D_4 = [4, \infty[\setminus \{2, 3\}$

5) $f_5(x) = 13\sqrt{5-x} + 18\log_5(x-7) + \frac{3}{x}$

$D_5 = \emptyset$

 $\emptyset$ is named the *empty set*

$5-x \geq 0 \quad \text{and} \quad x-7 \geq 0 \quad \text{then} \quad x \leq 5 \quad \text{AND} \quad x \geq 7 \Rightarrow \text{impossible!}$

Question 2

Given $f(x) = \frac{4x}{x+1}$ and $g(x) = \frac{x-1}{4}$

1) $(f \circ g)(x) = \frac{4\left(\frac{x-1}{4}\right)}{\left(\frac{x-1}{4}\right)+1} = \frac{x-1}{\frac{x-1}{4}+1} = \frac{x-1}{\frac{x+3}{4}} = (x-1)\frac{4}{x+3} = \frac{4(x-1)}{x+3}$

2) $(f \cdot g)(x) = \frac{4x}{x+1} \cdot \frac{x-1}{4} = \frac{x(x-1)}{x+1}$

3) Bonus :

$(f \circ g)(x) = (f \cdot g)(x) \Leftrightarrow \frac{4(x-1)}{x+3} = \frac{x(x-1)}{x+1} \Leftrightarrow \frac{4}{x+3} = \frac{x}{x+1} \Leftrightarrow 4(x+1) = x(x+3) \Leftrightarrow x^2 - x - 4 = 0$

$\Leftrightarrow \begin{matrix} x_1 \\ x_2 \end{matrix} = \frac{-1 \pm \sqrt{17}}{2}$