Problem 1

The diagram below shows a circle, centre O, with a radius 12 cm. The chord AB subtends at an angle of 75° at the centre. The tangents to the circle at A and at B meet at P.

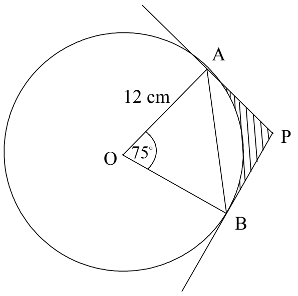


diagram not to scale

(a) Using the cosine rule, show that the length of AB is $12\sqrt{2(1-\cos 75^\circ)}$.

(b) Find the length of BP.

(c) Hence find

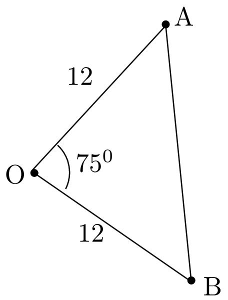
(i) the area of triangle OBP;

(ii) the area of triangle ABP.

(d) Find the area of **sector** OAB.

(e) Find the area of the shaded region.

a)



$$AB^2 = 2\times 12^2 - 2\times 12\times 12\cos(75^0)$$
$$=12^2\times 2(1-\cos(75^0))$$

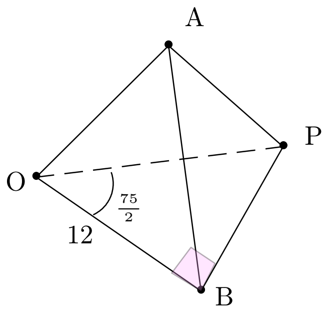
then

$$AB=12\sqrt{2((1-\cos(75^0)}$$

the numerical value is :

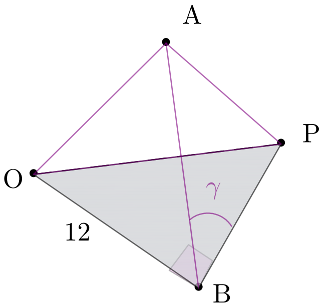
$$AB \cong \boxed{14.61\text{ cm}}$$

b)



$$\tan\left(\frac{75}{2}\right)=\frac{PB}{OB}$$
$$\Rightarrow$$
$$PB=12\tan(\tfrac{75}{2})$$
$$=\boxed{9.2\text{cm}}$$

b)



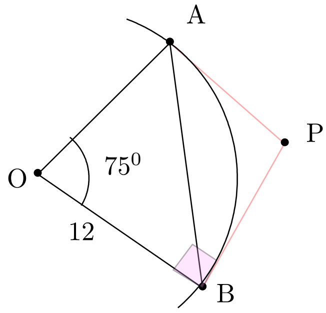
i) Area of OBP

$$=\tfrac{1}{2}OB\times BP$$
$$=\tfrac{1}{2}12\times 9.2=\boxed{55.2\text{cm}^2}$$

ii) Area of ABP

$$=\tfrac{1}{2}AB\times BP\sin(\gamma)$$
$$=\tfrac{1}{2}14.61\times 9.2\sin\left(180-90-\left(180-90-\frac{75}{2}\right)\right)$$
$$=67.2\sin\left(\frac{75}{2}\right)=67.2\sin(15)=\boxed{40.9\text{cm}^2}$$

d) and e)



Area sector OAB

$$=\tfrac{1}{2}\left(\frac{75}{180}\pi\right)12^2$$
$$=\boxed{94.25\text{cm}^2}$$

Area shaded area

$$=2\times \text{area(OBP)}$$
$$-\text{Area sector OAB}$$
$$=110.4-94.25$$
$$=\boxed{16.15\text{cm}^2}$$

Problem 2

In the following diagram, O is the centre of the circle and (AT) is the tangent to the circle at T.

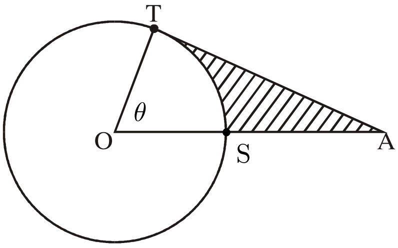


Diagram not to scale

If OA = 12 cm, and the circle has a radius of 6 cm, find the area of the shaded region.

We start by finding the are of the triangle AOT and the area of the circular sector OTS

Then we can get the area of the shaded region by subtraction.

Step 1: For knowing the are of the triangle AOT we need the length TA.

We can get it by Pythagoras (because the triangle is rectangle at T)

$TA^2 = OA^2 - OT^2 \Rightarrow TA = \sqrt{12^2 - 6^2} = \sqrt{180} = 6\sqrt{3}\,\text{cm}$

Then the area of the triangle AOT is $\frac{OT \times TA}{2} == 18\sqrt{3}\text{cm}^2$

Step 1: For knowing the area of the circular sector OTS we use the formula : $\mathcal{A} = \frac{1}{2}\theta r^2$

but for that we need to know the angle θ. $\tan(\theta) = \frac{TA}{OT} \Rightarrow \theta = \arctan(\sqrt{3}) = 60^0 = \frac{\pi}{3}\text{rad}$.

Then the area of the circular sector OTS is $\mathcal{A} = \frac{1}{2}\frac{\pi}{3}6^2 = 6\pi\text{cm}^2$

Conclusion : the area of the shaded region is $18\sqrt{3} - 6\pi = \boxed{6\,(3\sqrt{3} - \pi)}$ $(\cong 12.33\text{cm}^2)$

Problem 3

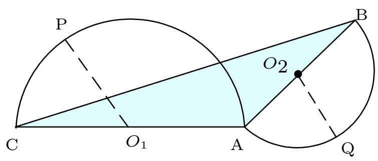
The length of segment BC is 21 cm

The *length* of the *semi-circle* of radius O_1P is 22 cm

The *surface area* of the *semi-circle* of radius O_2Q is 39.3 cm^2

i) Find the angle $\widehat{BAC}$

ii) Find the surface area of the triangle ABC.



i) $\frac{1}{2}(2\pi O_1P) = 22 \Rightarrow O_1P = \frac{22}{\pi} = 7.00\text{cm}$

$\frac{1}{2}\pi(O_2Q)^2 = 39.3 \Rightarrow O_2Q = \sqrt{\frac{78.6}{\pi}} = 5.00\text{cm}$

then as $BC^2 = AC^2 + AB^2 - 2\,AC\,AB\cos(\widehat{BAC})$: $\cos(\widehat{BAC}) = \frac{21^2 - (2\times 7)^2 - (2\times 5)^2}{-2(2\times 7)(2\times 5)} = \frac{145}{-280} = -\frac{29}{54}$

$\Rightarrow BAC = \arccos\left(-\frac{29}{54}\right) = \boxed{122.48^0}$

ii) $\mathcal{A} = \frac{1}{2}AC \times AB\sin(122.48^0) = \boxed{59.0\text{cm}^2}$