PreTest 4b

subject: *Trigonometry I*

Mathematics - IB_1 - | Tuesday 22 Jan 2025

Total : / 36 marks

ANSWERS

Problem 1

Assuming θ is in the *fourth* sector, and $\sin(\theta)=-\sqrt{\frac{3}{7}}$, give the exact expression for

i) $\cos(\theta)$, **ii)** $\tan(\theta)$, **iii)** $\cos(2\theta)$, **iv)** $\sin(2\theta)$, **v)** $\tan(2\theta)$

i) $\boxed{\sqrt{\frac{4}{7}}=2\frac{\sqrt{7}}{7}}$ **ii)** $-\frac{\sqrt{3}}{2}$ **iii)** $\boxed{\frac{1}{7}}$ **iv)** $2\left(-\sqrt{\frac{3}{7}}\right)\left(\sqrt{\frac{4}{7}}\right)=\boxed{-\frac{4\sqrt{3}}{7}}$ **v)** $\boxed{-4\sqrt{3}}$

Problem 2 (without calculator) *[13 marks]*

We consider the following trigonometric equations:

1) $\sin(3x)=\frac{\sqrt{2}}{2}$ *for* $0\leqslant x<360^0$ *[4 marks]*

As $\frac{\sqrt{2}}{2}>0$, then angle $3x$ is either in the *first* or in the *second* sector.

In the *first* : $3x=45+k360 \Rightarrow \boxed{x=15+k120}$ in the *second* : $3x=135+k360$ $\boxed{x=45+k120}$

that means $\boxed{15^0, 45^0, 135^0, 165^0, 255^0 \text{ and } 2^0}$ are *six distinct solutions* between 0 and 360^0

2) $\cos(4x)=\frac{\sqrt{3}}{2}$ *for* $0\leqslant x<2\pi$ *(*radian*)* *[4 marks]*

$4x=\frac{\pi}{6}$ $(+2k\pi)$ or $4x=\frac{5\pi}{6}$ $(+2k\pi)$

$\Rightarrow x=\boxed{\frac{\pi}{24}+k\frac{\pi}{2}}$ or $x=\boxed{\frac{5\pi}{24}+k\frac{\pi}{2}}$

therefore $S=\left\{\frac{\pi}{24},\frac{5\pi}{24},\frac{13\pi}{24},\frac{17\pi}{24},\frac{25\pi}{24},\frac{29\pi}{24},\frac{37\pi}{24},\frac{41\pi}{24}\right\}$: *height distinct solutions* between 0 and 2π

3) $6\cos(2x)-4\cos^2(x)=0$, for $0\leqslant x<3\pi$ *(radian)* *[5 marks]*

$S=\left\{\frac{\pi}{6}+2\pi k\right\}\cup\left\{\frac{5\pi}{6}+2\pi k\right\}\cup\left\{\frac{7\pi}{6}+2\pi k\right\}\cup\left\{\frac{11\pi}{6}+2\pi k\right\}$

As we want only $0\leqslant x<3\pi$, we get : $S=\left\{\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6},\frac{13\pi}{6},\frac{17\pi}{6}\right\}$

Problem 3 (with calculator) *[8 marks]*

Consider the trigonometric equation $5\cos(2\theta)=3(\cos(\theta)+1)-4$

i) It can be written as

$a\cos^2(\theta)+b\cos(\theta)+c=0$ $\left(=10\cos^2(\theta)-3\cos(\theta)-4\right)$ *[4 marks]*

with $a=10,\ b=-3,\ c=-4,\ \Delta=49$ $x=\frac{-3\pm 7}{20}$

ii) The set S of solutions of this equation, for $0\leqslant x<2\pi$ (radian) is *[4 marks]*

$S=\left\{\frac{\pi}{3}+2\pi k\right\}\cup\left\{\frac{5\pi}{3}+2\pi k\right\}\cup\left\{\frac{5\pi}{3}+2\pi k\right\}\cup\left\{\arccos\left(\frac{1}{5}\right)+2\pi k\right\}\cup\left\{2\pi-\arccos\left(\frac{1}{5}\right)+2\pi k\right\}$