Mathematics - IB_1 -

PreTest 4a

subject: *Trigonometry I*

Tuesday 22 Jan 2025

ANSWERS

Total : / 36 marks

For solving these problems, you may have to use some of theses following *trigonometric identities*

$\cos^2(x)+\sin^2(x)=1$	$\sin(2x)=2\sin(x)\cos(x)$
$\tan(x)=\frac{\sin(x)}{\cos(x)}$	$\cos(2x)=\cos^2(x)-\sin^2(x)$

Problem 1

[6 marks]

1) The *exact value* of $\cos(\frac{\pi}{6})$ is $\boxed{\frac{\sqrt{3}}{2}}$

2) Using the first relation in the frame above we can know $\cos(x)$ in terms of $\sin(x)$

3) Assuming $\sin(\theta)=\frac{4}{5}$, by the first identity: $\cos^2(x)=1-\sin^2(x)$

then $\cos(x)=\pm\sqrt{1-\sin^2(x)}\ =\pm\frac{3}{5}$

hence the two possible values for $\tan(\theta)$ are $\frac{\frac{4}{5}}{\pm\frac{3}{5}}=\boxed{\pm\frac{4}{3}}$

Problem 2

[10 marks]

1) Assuming θ is in the *second* sector, and $\sin(\theta)=\frac{15}{17}$, give the exact expression for

i) $\cos(\theta)$, **ii)** $\cos(2\theta)$, **iii)** $\sin(2\theta)$, **iv)** $\tan(\theta)$, **v)** $\tan(2\theta)$

i) $\boxed{-\frac{8}{17}}$ **ii)** $(-\frac{8}{17})^2-(\frac{15}{17})^2=\boxed{-\frac{161}{289}}$ **iii)** $2(-\frac{8}{17})(\frac{15}{17})=\boxed{-\frac{240}{289}}$ **iv)** $\boxed{-\frac{8}{15}}$ **v)** $\boxed{\frac{240}{161}}$

2) Assuming θ is in the *third* sector, and $\tan(\theta)=\sqrt{\frac{17}{8}}$, give the exact expression for

i) $\cos(\theta)$, **ii)** $\sin(\theta)$, **iii)** $\cos(2\theta)$, **iv)** $\sin(2\theta)$, **v)** $\tan(2\theta)$

$\frac{\sin^2(\theta)}{\cos^2(\theta)}=\frac{17}{8}$ and $17+8=25$ $\Rightarrow \cos(\theta)=\pm\sqrt{\frac{8}{25}}$ and $\sin(\theta)=\pm\sqrt{\frac{17}{25}}$

as θ is in the *third*, $\boxed{\cos(\theta)=-\frac{2\sqrt{2}}{5}}$ $\boxed{\sin(\theta)=\frac{\sqrt{17}}{5}}$

$\boxed{\cos(2\theta)=\left(-\frac{2\sqrt{2}}{5}\right)^2-\left(\frac{\sqrt{17}}{5}\right)^2=-\frac{9}{25}}$ $\boxed{\sin(2\theta)=2\left(-\frac{2\sqrt{2}}{5}\right)\left(\frac{\sqrt{17}}{5}\right)=\frac{-4\sqrt{34}}{25}}$

you can easily verify that $\cos^2(2\theta)+\sin^2(2\theta)$

finally $\boxed{\tan(2\theta)=\frac{\sin(2\theta)}{\cos(2\theta)}=\frac{4\sqrt{34}}{9}}$

Problem 3 : *Please see next page !*

Problem 4

[8 marks]

Consider the trigonometric equation $3\cos(2\theta)+4\cos(\theta)+5=-1$

i) Using the identity for $\cos(2x)$ (here $x=\theta$) and the first identity, we transform the equation in: $3\,(\cos^2(\theta)-\sin^2(\theta))+4\cos(\theta)+5=-1$ $3\,(2\cos^2(\theta)-1)+4\cos(\theta)+5=-1$

then $a\cos^2(\theta)+b\cos(\theta)+c=0$ with $\boxed{a=6, b=4, c=3}$

ii) Hence $\Delta=b^2-4ac=16-72<0$ hence $\boxed{S=\varnothing}$: no x solution for $0\leqslant x<2\pi$

Problem 3 ***[14 marks]***

Solve the following trigonometric equations:

1) $\cos(x) = \frac{1}{2}$

As $\frac{1}{2} > 0$, x is an angle either in the first or in the third sector

In the first : $\boxed{x = \frac{\pi}{3}}$ $(+2k\pi)$ in the third : $\boxed{x = \frac{5\pi}{3}}$ $(+2k\pi)$ S

2) $\cos(2x) = \frac{1}{2}$

As $\frac{1}{2} > 0$, x is an angle either in the *first* or in the *third* sector

In the *first* : $2x = \frac{\pi}{3} + 2k\pi \Rightarrow \boxed{x = \frac{\pi}{6} + k\pi}$ in the *third* : $2x = \frac{\pi}{3} + 2k\pi$ $\boxed{x = \frac{5\pi}{6} + k\pi}$

that means $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ are *four distinct solutions* between 0 and 2π

3) $\sin(x) = \frac{\sqrt{2}}{2}$

As $\frac{\sqrt{2}}{2} > 0$, x is an angle either in the *first* or in the *second* sector

In the first : $\boxed{x = \frac{\pi}{4}}$ $(+2k\pi)$ in the third : $\boxed{x = \frac{3\pi}{4}}$ $(+2k\pi)$

4) $\sin(4x) = \frac{\sqrt{2}}{2}$

$4x = \frac{\pi}{4}$ $(+2k\pi)$ or $4x = \frac{3\pi}{4}$ $(+2k\pi)$

$\Rightarrow x = \boxed{\frac{\pi}{16} + k\frac{\pi}{2}}$ or $x = \boxed{\frac{3\pi}{16} + k\frac{\pi}{2}}$

that means $\frac{\pi}{16}, \frac{3\pi}{16}, \frac{9\pi}{16}, \frac{11\pi}{16}, \frac{17\pi}{16}, \frac{19\pi}{16}, \frac{25\pi}{16}$ and $\frac{27\pi}{16}$ are *height distinct solutions* between 0 and 2π

5) $2\cos(x)^2 - 3\sin(x) = 0$ *[5 marks]*

$\Rightarrow 2\,(1 - \sin(x)^2) - 3\sin(x) = 0 \Rightarrow -2\sin(x)^2 - 3\sin(x) + 2 = 0$

then $\sin(x) = \frac{3 \pm \sqrt{25}}{-4} = \begin{cases} -2 \\ \frac{1}{2} \end{cases}$ (notice sin *cannot* be equal to -2)

$\Rightarrow \sin(x) = \frac{1}{2} \Rightarrow$ the first two positive solutions are: $\boxed{x = \frac{\pi}{6}\text{rad}\,,\, x = \frac{5\pi}{6}\text{rad}}$ $\boxed{x = 30^0\,,\, x = 150^0}$

6) $6\cos x - 4\sin^2 x = 0$, $0 \leqslant x < 3\pi$ *(radian)* *[5 marks]*

$6\cos x - 4\,(1 - \cos(x)^2) = 0 \Rightarrow 4\cos(x)^2 + 6\cos(x) - 4 = 0$

then $\cos(x) = \frac{-6 \mp \sqrt{100}}{8} = \begin{cases} -2 \\ \frac{1}{2} \end{cases}$ (notice cos *cannot* be equal to -2)

$\Rightarrow \cos(x) = \frac{1}{2} \Rightarrow \boxed{x = \frac{\pi}{3}\text{ rad or } x = \frac{5\pi}{3}\text{ rad or } x = \frac{7\pi}{3}\text{ rad}}$ assuming $0 \leqslant x < 3\pi$

Bonus :

$4\sin^2(2x) = 3 \quad \Rightarrow \sin(2x) = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow 2x = \frac{\pi}{3} + k2\pi$ or $2x = \frac{2\pi}{3} + k2\pi$ or $2x = \frac{4\pi}{3} + k2\pi$ or $2x = \frac{5\pi}{3} + k2\pi$

$\Rightarrow x = \frac{\pi}{6} + k\pi$ or $x = \frac{\pi}{3} + k\pi$ or $x = \frac{2\pi}{3} + k\pi$ or $x = \frac{5\pi}{6} + k\pi$

for $0 \leqslant x < 360^0$: $x \in \{30^0, 60^0, 120^0, 150^0, 180^0, 210^0, 240^0, 300^0, 330^0, 360^0\}$