Mathematics - IB_1 -

Pre- Test 9

(For February 2016)

sujet: *Trigonometry II*

Problem 1 *[11 marks]*

A)

Tot : / 34 marks

The following diagram shows a triangle ABC.

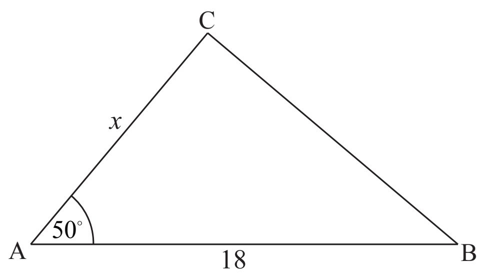


diagram not to scale

The area of triangle ABC is 80 cm², AB = 18 cm, AC = x cm and $B\hat{A}C = 50^\circ$.

(a) Find x. $\frac{1}{2}18 \times x \times \sin(50) = 80 \Rightarrow x = \frac{80}{9\sin(50)} = 11.6\text{cm}$ *[3 marks]*

(b) Find BC. $BC = \sqrt{18^2 + 11.6^2 - 2 \times 18 \times 11.6 \times \cos(50)} = 13.8\text{cm}$ *[3 marks]*

B)

Let us consider a triangle ABC quite similar to the precedant above.

Point A lies on the horizontal dashed line.

Assuming BC=13.8 cm AC = x , and $\beta = 40.2^0$ find two possibles positions for A.

$\frac{13.78}{\sin(\alpha)} = \frac{11.6}{\sin(\beta)} \Rightarrow \sin(\alpha) = \frac{13.78}{11.6}\sin(40.2) = 0.767$

Therefore either $\alpha = \arcsin(0.767) = 50^0$ or $\alpha = 180 - 50 = 130^0$

Notice : That is typically an example of *ambigus case*.

The two possible positions for A

are given by the relation $\frac{11.6}{\sin(40.2)} = \frac{AB}{\sin(180 - 40.2 - \alpha)}$

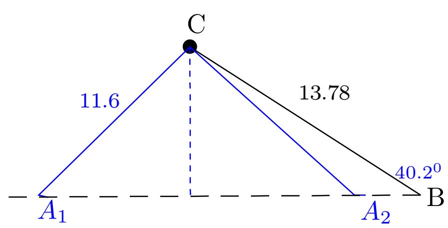
Then :

$AB = \frac{11.6}{\sin(40.2)}\sin(139.8 - \alpha)$ with $\alpha = 50^0$ or 130^0.

That give two values for AB (one for each value of β)

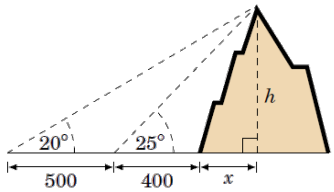
– for $\alpha = 50^0$: $\boxed{AB = 18\text{cm}}$ (like in part A) (A_1)

– for $\alpha = 130^0$: $\boxed{AB = 3.05\text{cm}}$ (A_2)



Problem 2 *[9 marks]*

A small montain of height h si observed trom two position with two different angles as show on the picture below.



$\tan(20) = \frac{h}{900+x}$ and $\tan(25) = \frac{h}{400+x}$

Then $(900+x)\tan(20) = (400+x)\tan(25)$

$\Rightarrow x = \frac{900\tan(20) - 400\tan(25)}{\tan(25) - \tan(20)} = 154m$

and $\boxed{h = 258m}$

Problem 3 *[8 marks]*

i) Solve the trigonometric equation $\sin(x-1) = 2 - \sin(x-1)$

ii) Solve the trigonometric equation $\cos(2x) = \cos(x) + \frac{1}{2}$

Answers: (i) Equivalent to $\sin(x-1) = 1$

$\Rightarrow (x-1) = \frac{\pi}{2} + 2k\pi \quad S = \left\{1 + \frac{\pi}{2} + 2k\pi\right\}_{k\in\mathbb{Z}}$

(ii) $2\cos^2(x) - 1 = \cos(x) + \frac{1}{2}$ ... (made in class Friday Jan.29)

Problem 4 *[6 marks]*

By using trigonometric identities, show that : $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

A god start is the relation $\frac{1}{\tan(2x)} = \frac{1-\tan^2(x)}{2\tan(x)} = \frac{1}{2\tan(x)} - \frac{\tan^2(x)}{2\tan(x)} = \ldots.$

B.Bratschi. 30.1.2016